Solving rational expectations models with first order perturbation: what Dynare does

EBERHARD KARLS JIVERSITÄT TÜBINGEN



Willi Mutschler





DSGE Model Framework

$$E \left[f_{\theta} \left(y_{t-1}, y_t \right) \right]$$
$$\mathcal{U}_{S} \sim$$

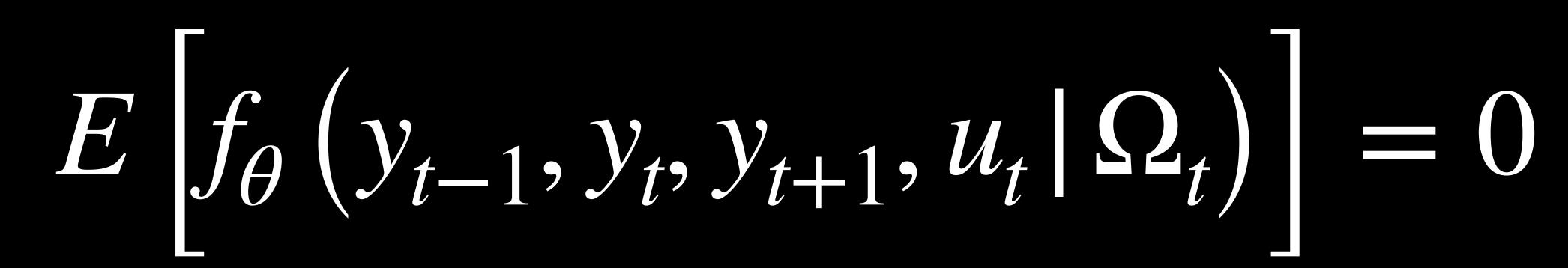
- $t, s \in \mathbb{T}$: discrete time set, typically \mathbb{N} or \mathbb{Z}
- *y_t*: *n* endogenous variables (declared in var block)
- $u_t: n_u$ exogenous variables (declared in *varexo* block)
- Σ_{μ} : covariance matrix of invariant distribution of exogenous variables (declared in *shocks*) block)
- θ : n_{θ} model parameters (declared in *parameters* block)
- *f*: *n* model equations (declared in *model* block)
- f_{θ} is a continuous non-linear function indexed by a vector of parameters θ

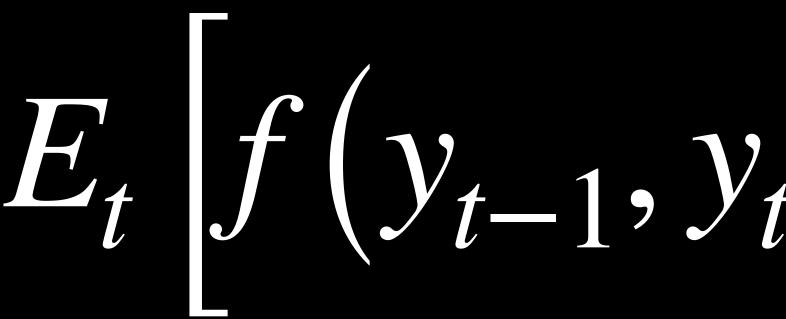
 $\left[y_{t}, y_{t+1}, u_{t} | \Omega_{t} \right] = 0$ $WN(0, \Sigma_{u})$

 $E\left[f_{\theta}\left(y_{t-1}, y_{t}, y_{t+1}, u_{t} \mid \Omega_{t}\right)\right] = 0$ $u_{s} \sim WN(0, \Sigma_{u})$

Rational Expectations

- information set includes model equations *f*, value of parameters θ, value of current state y_{t-1}, value of current exogenous variables u_t, invariant distribution (but not values!) of future exogenous variables u_{t+s}
- Ω_t : information set (*filtration*, i.e. $\Omega_t \subseteq \Omega_{t+s} \forall s \ge 0$)
- $\Omega_t = \{f, \theta, y_{t-1}, u_t, u_{t+s} \sim N(0, \Sigma)\} \text{ for all } t \in \mathbb{T}, s > 0$
- $E[\cdot | \Omega_t]$: conditional expectation operator, typically we use shorthand E_t





$E_{t}\left[f(y_{t-1}, y_{t}, y_{t+1}, u_{t})\right] = 0$

Perturbation approach

- Step 1: Introduce perturbation parameter
 - scale u_t by a parameter $\sigma \ge 0$: $u_t = \sigma \varepsilon_t$ with $\varepsilon_t \sim WN(0, \Sigma_{\varepsilon})$
 - note that this implies $\Sigma_{\mu} = \sigma^2 \Sigma_e$
 - σ is called the *perturbation parameter*
 - non-stochastic, i.e. static model: $\sigma = 0$
 - stochastic, i.e. dynamic model: $\sigma > 0$

Step 2: *define* dynamic solution

- invariant mapping between y_t and (y_{t-1}, u_t) :
- $g(\cdot)$ is called the *policy-function* or *decision* rule
- $g(\cdot)$ is unknown, i.e. we need to solve a functional equation

 $y_t = g(y_{t-1}, u_t, \sigma)$

Idea: Maybe we can get *g* from $E_t \left[f(y_{t-1} - F_{t-1}) + y_t \right] = g(y_{t-1}, u_t, \sigma)$

 $y_{t+1} = g(y_t, u_{t+1}, \sigma) = g(g(y_{t-1}, u_t, \sigma), u_{t+1}, \sigma)$

$$(y_{t-1}, y_t, y_{t+1}, u_t) = 0?$$



$= f \left(y_{t-1}, g(y_{t-1}, u_t, \sigma), g(g(y_{t-1}, u_t \sigma), u_{t+1}, \sigma), u_t \right)$

General idea

Rewrite dynamic model: $f(y_{t-1}, y_t, y_{t+1}, u_t)$

 $\equiv F(y_{t-1}, u_t, u_{t+1}, \sigma)$

Perturbation is based on the *implicit function theorem*:

$$E_t F(y_{t-1}, u_t, u_{t+1})$$

 $_{+1}, \sigma) = 0$ [known]

implicitly defines

 $g(y_{t-1}, u_t, \sigma)$ [unknown]

We know how to solve for the non-stochastic ($\sigma = 0$) steady-state \bar{y} by solving the *static* model:

$f(\bar{y}) \equiv f(\bar{y}, \bar{y}, \bar{y}, 0) = F(\bar{y}, 0, 0, 0) = 0$

which provides us with the non-stochastic steady-state for \bar{y}

Even though we do not know $g(\cdot)$ explicitly, we do know its value at \bar{y} :

- $\bar{y} = g(\bar{y}, 0, 0)$

Taylor approximation of g

 $y_t = g(y_{t-1}, u_t, \sigma)$

Let's approximate $g(\cdot)$ around \bar{y} with a 1st order Taylor expansion:

$$y_t \approx \bar{y} + \left[\frac{\partial g(\bar{y}, 0, 0)}{\partial y'_{t-1}}\right] (y_{t-1} - \bar{y}) + \left[\frac{\partial g(\bar{y}, 0, 0)}{\partial u'_t}\right] (u_t - 0) + \left[\frac{\partial g(\bar{y}, 0, 0)}{\partial \sigma}\right] (\sigma - 0)$$

Some progress: instead of an infinite unknown number of parameters for *g*, we have now only *three* unknown matrices

Taylor approximation of g

But: how do we obtain these?

 \blacktriangleright Let's approximate $F(\cdot)$ around \bar{y} with a 1st order Taylor expansion!

More Notation

 $u := u_t, u_+ := u_{t+1}$

 $y_{-} := y_{t-1}, y_{0} := y_{t}, y_{+} := y_{t+1}$ $r := \begin{pmatrix} y_{-} \\ u \\ u_{+} \\ \sigma \end{pmatrix} \qquad z := \begin{pmatrix} y_{-} \\ y \\ y_{+} \\ u \end{pmatrix} = \begin{pmatrix} y_{-} \\ g(y_{-}, u, \sigma) \\ g(g(y_{-}, u, \sigma), u_{+}, \sigma) \\ u \end{pmatrix}$

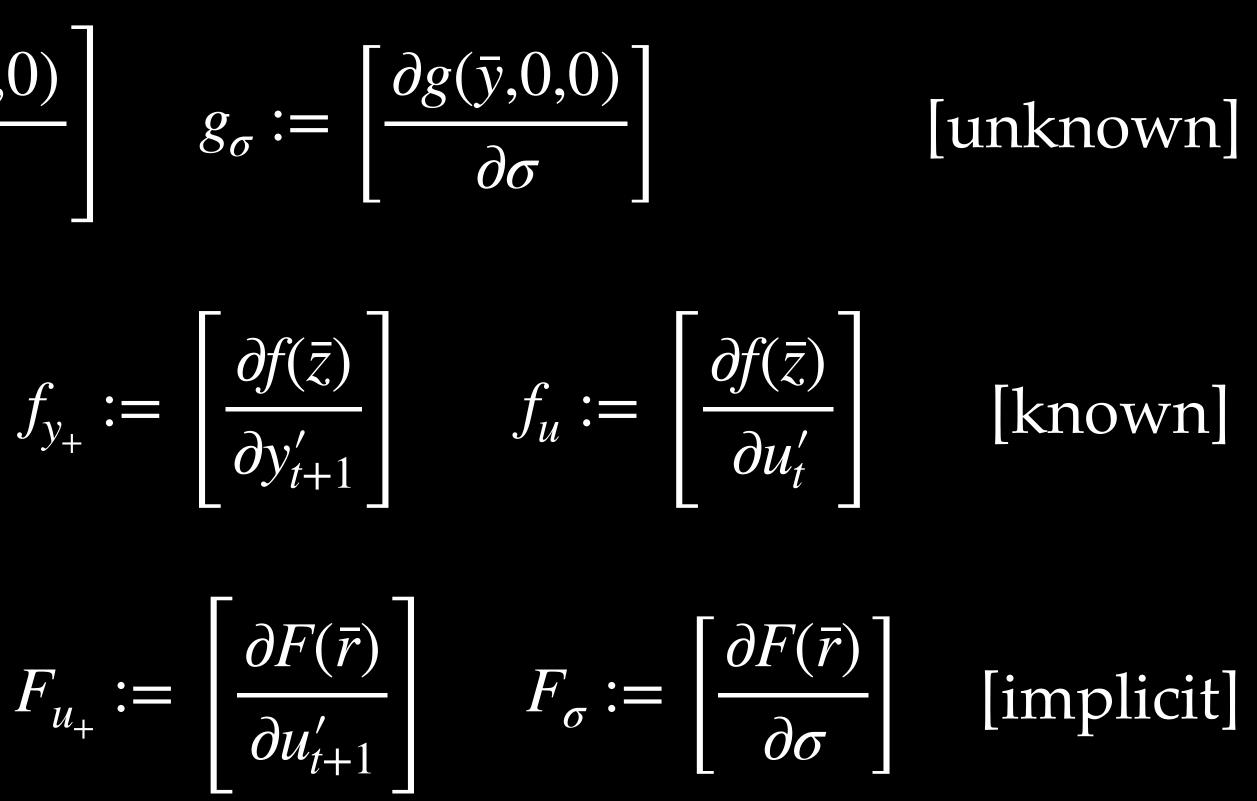
Notation Jacobian Matrices

$$g_{y} := \begin{bmatrix} \frac{\partial g(\bar{y}, 0, 0)}{\partial y'_{t-1}} \end{bmatrix} \qquad g_{u} := \begin{bmatrix} \frac{\partial g(\bar{y}, 0, 0)}{\partial u'_{t}} \end{bmatrix}$$

$$f_{y_{-}} := \begin{bmatrix} \frac{\partial f(\bar{z})}{\partial y'_{t-1}} \end{bmatrix} \qquad f_{y_{0}} := \begin{bmatrix} \frac{\partial f(\bar{z})}{\partial y'_{t}} \end{bmatrix} \qquad \bar{J}$$

$$F_{y} := \begin{bmatrix} \frac{\partial F(\bar{r})}{\partial y'_{t-1}} \end{bmatrix} \qquad F_{u} := \begin{bmatrix} \frac{\partial F(\bar{r})}{\partial u'_{t}} \end{bmatrix}$$

All derivatives are evaluated at the non-stochastic steady-state



Taylor approximation of F

Let's approximate $F(r) = F(y_{t-1}, u_t, u_{t+1}, \sigma)$ around \bar{r} at 1st order:

with $\hat{y} = (y_{-} - \bar{y})$, $\hat{u} = (u - 0) = u$, $\hat{u}_{+} = (u_{+} - 0) = \sigma \varepsilon_{+}$, $\hat{\sigma} = (\sigma - 0) = \sigma$

 $F(r) \approx F(\bar{r}) + F_v \hat{y}_- + F_u \hat{u} + F_{u_\perp} \hat{u}_+ + F_\sigma \hat{\sigma}$

Taylor approximation of F

Our model implies that $E_t F(r) = 0$, so let's use this on the first-order approximation:

$0 = E_t F(r) \approx 0 + F_v \hat{y}_-$

$0 \approx F_v \hat{y}_- + F_u u$

$$F_y = 0$$
 and $F_u = 0$ and $F_\sigma + F_{u_+} E_t \varepsilon_+ = 0$

$$+ F_{u} u + F_{u_{+}} E_{t} \sigma \varepsilon_{+} + F_{\sigma} \sigma$$

$$+ \left(F_{\sigma} + F_{u_{+}} E_{t} \varepsilon_{+}\right) \sigma$$

Insight: this equation needs to be satisfied for <u>any</u> value of $\hat{y}_{,u}$ and σ ; hence:

Taylor approximation of *F*

We have 3 (multivariate) equations:

$$F_y = 0$$
 and $F_u =$

to recover three unknown matrices

- $\bullet g_y \text{ from } F_y = 0$
- g_u from $F_u = 0$
- g_{σ} from $F_{\sigma} + F_{u_{+}}E_{t}\varepsilon_{+} = 0$

0 and $F_{\sigma} + F_{u_+} E_t \varepsilon_+ = 0$

Recovering g_{σ}

First order derivative with respect to σ yields: $F_{\sigma} = f_{y_0} g_{\sigma} + f_{y_+} (g_y g_{\sigma} + g_{\sigma})$

First order derivative with respect to u_{\perp} yields:

Recovering go $F = f \quad y_{-}, g(y_{-}, u, \sigma), g(\overline{g(y_{-}, u, \sigma)}, u_{+}, \sigma), u_{-}, u_{-}, \sigma)$

 $F_{u_+} = f_{y_+} g_u$

Recovering g_{σ}

 $\int_{y_0} g_{\sigma} + f_{y_+} (g_x g_{\sigma} + g_{\sigma}) + f_{y_+} g_u E_t \varepsilon_+ = 0$ $\Leftrightarrow \mathbf{g}_{\sigma} = -\left(f_{y_0} + f_{y_+}\mathbf{g}_x + f_{y_+}\right)^{-1} f_{y_+}\mathbf{g}_u E_t \mathbf{\varepsilon}_+$

Of course, we know that $E_t \varepsilon_{t+1} = 0$, which implies:

 $g_{\sigma} = 0$

Certainty Equivalence $g_{\sigma} = 0$

take into account the effect of future uncertainty when optimizing

$$\hat{y}_t = g_y \hat{y}_{t-1}$$

Future uncertainty does not matter for the decision rules of the agents!

we can break it with e.g. higher-order perturbation approximation

- When we derived the optimality conditions (aka model equations) agents do
- BUT: the policy function is independent of the size of the stochastic innovations:

$$+ g_u u_t + 0 \cdot \sigma$$

- Certainty equivalence is a result of the first-order perturbation approximation,

Recovering g_u

Recovering g₁ $F = f \left[\begin{array}{c} y_{-}, g(y_{-}, u, \sigma) \\ y_{0} \end{array} \right], g\left(\begin{array}{c} y_{0} \\ g(y_{-}, u, \sigma) \\ y_{+} \end{array} \right), u_{+}, \sigma \right), u$

First order derivative with respect to *u* yields:

$$F_{\mu} = f_{y_0}g$$

$$F_u = 0$$
 implies:

$$g_{\boldsymbol{u}} = -\left(f_{\boldsymbol{y}_0} + f_{\boldsymbol{y}_+}g_{\boldsymbol{y}}\right)^{-1}f_{\boldsymbol{u}}$$

 $g_{u} + f_{y_{+}}g_{y}g_{u} + f_{u}$

Recovering g_u

$$g_u = -\left(j\right)$$

This is a linear equation which requires computing an inverse involving g_v

Therefore: once we know $g_{y'}$ we can easily compute $g_{u'}$.

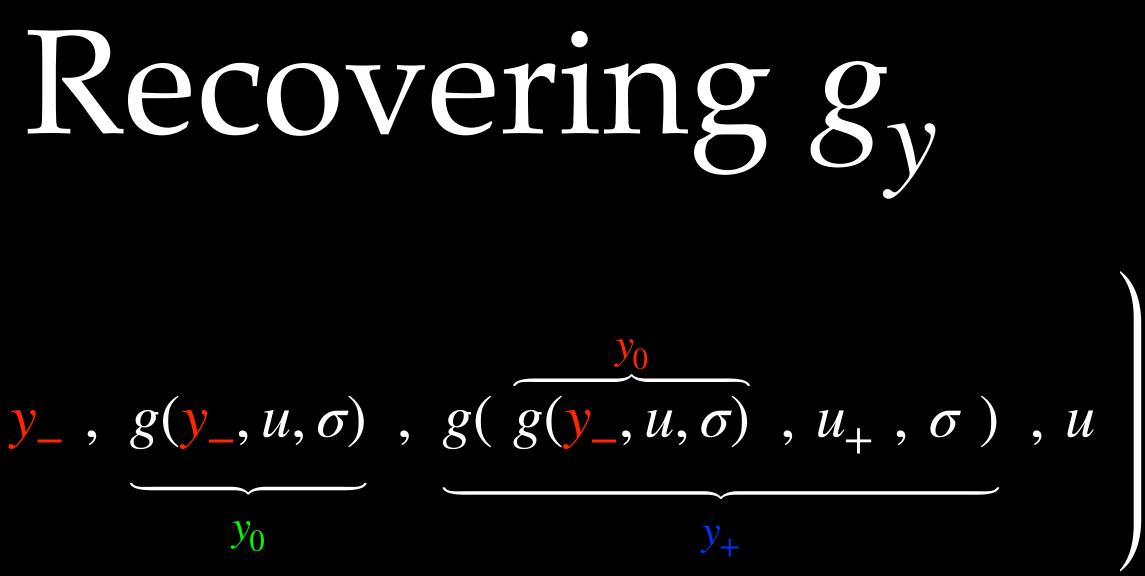
 $\left(f_{y_0} + f_{y_+}g_y\right)^{-1} f_u$

Recovering g_y

$$F = f \quad y_{-}, g(y_{-}, u, \sigma)$$

First order derivative with respect to y_ and setting it to zero yields: $F_{\mathbf{y}} = f_{\mathbf{y}} + j$

This is a *quadratic equation*, but the unknown g_v is a matrix!



$$f_{y_0}g_{\mathbf{y}} + f_{y_+}g_{\mathbf{y}}g_{\mathbf{y}} \stackrel{!}{=} 0$$

- It is generally impossible to solve this equation analytically, but there are several ways to deal with this as this boils down to solving so-called *Linear Rational Expectations Models*

Linear Rational Expectations Model

Re-consider original dynamic model:

Take first-order Taylor expansion:

$$f_{y_{-}}\hat{y}_{t-1} + f_{y_{0}}\hat{y}_{t} +$$

In the literature this is known as a *Linear Rational Expectations Model*

 $E_t f(y_{t-1}, y_t, y_{t+1}, u_t) = 0$

$+ f_{y_{\perp}} E_t \hat{y}_{t+1} + f_u u_t = 0$

Linear Rational Expectations Model

$f_{y_{t-1}} + f_{y_{0}} \hat{y}_{t} +$

Using the first-order policy function:

$$E_t \hat{y}_{t+1} = g_y \hat{y}_t + g_u E_t u_{t+1} = g_y (g_y \hat{y}_{t-1} + g_u u_t) = g_y g_y \hat{y}_{t-1} + g_y g_u u_t$$

Rewriting the above equation we see the connection to perturbation:

$$(f_{y_{-}} + f_{y_{0}}g_{y} + f_{y_{+}}g_{y}g_{y}) \hat{y}_{t-1} = -(f_{y_{0}}g_{u} + f_{y_{+}}g_{y}g_{u} + f_{u}) u_{t} = 0$$

$$F_y=0$$

$$- f_{y_{+}} E_{t} \hat{y}_{t+1} + f_{u} u_{t} = 0$$

 $\hat{\mathbf{y}}_t = g_y \hat{\mathbf{y}}_{t-1} + g_u u_t$

 $F_{\mu}=0$

Structural State-Space System

 $f_{y_{-}}\hat{y}_{t-1} + f_{y_{0}}\hat{y}_{t} + f_{y_{+}}E_{t}\hat{y}_{t+1} + f_{u}u_{t} = 0$ $\begin{pmatrix} 0 & f_{y_{+}} \\ I & 0 \end{pmatrix} \begin{pmatrix} \hat{y}_{t} \\ E_{t} \hat{y}_{t+1} \end{pmatrix} = \begin{pmatrix} -f_{y_{-}} & -f_{y_{0}} \\ 0 & I \end{pmatrix} \begin{pmatrix} \hat{y}_{t-1} \\ \hat{y}_{t} \end{pmatrix} + \begin{pmatrix} -f_{u} \\ 0 \end{pmatrix} u_{t}$:=D $:=Y_t$ $D \cdot Y_t = E \cdot Y_{t-1} + U_t$

D and E are by construction square matrices

:=E	$:= Y_{t-1}$	U_t

<u>IF</u> *D* is invertible, then:

Stable solution if and only if all Eigenvalues λ_i of $(D^{-1}E)$ are inside unit circle

Stability

$D \cdot Y_t = E \cdot Y_{t-1} + U_t$

$Y_{t} = (D^{-1}E)Y_{t-1} + D^{-1}U_{t}$

$= (D^{-1}E)^0 D^{-1}U_t + (D^{-1}E)^1 D^{-1}U_{t-1} + (D^{-1}E)^2 D^{-1}U_{t-2} + (D^{-1}E)^3 D^{-1}U_{t-3} + \dots$

Stability

REMINDER: Eigenvalue λ_i and corresponding eigenvector v_i of $(D^{-1}E)$ satisfy:

BUT: *D* is typically singular and non-invertible! THEREFORE: use Generalized Eigenvalues λ_i that satisfy:

SAME IDEA: stability only for $|\lambda_i| < 1$ (inside unit circle)

MATLAB: Lambda = eig(E,D)

- $\lambda_i v_i = (D^{-1}E)v_i$

 - $\lambda_i D v_i = E v_i$

Generalized Schur Decomposition

Eigenvalue is defined via a zero determinant of matrix pencil: $det(D + \lambda E) = 0$

So instead of inverse we'll use a Schur decomposition on matrix pencil:

Q is orthogonal: $Q' = Q^{-1}$ and Q'Q = QQ' = IZ is orthogonal: $Z' = Z^{-1}$ and Z'Z = ZZ' = I*T* is upper triangular and *S* is quasi-upper triangular

MATLAB: [S,T,Q,Z] = qz(E,D)

D = Q'TZ' and E = Q'SZ'

Generalized Eigenvalues

Stability: look at *Generalized Eigenvalues* of *D* and *E*:

which can be found on the diagonal of S and T:

If $T_{ii} = 0$, then: $S_{ii} < 0 \rightarrow \lambda_i = -\infty$ $S_{ii} > 0 \rightarrow \lambda_i = \infty$ and

 $\lambda_i D v_i = E v_i$

$$\lambda_i = \frac{S_{ii}}{T_{ii}}$$

 \mathbf{C}

Structural State-Space System $\begin{pmatrix} 0 & f_{y_{+}} \\ I & 0 \end{pmatrix} \begin{pmatrix} \hat{y}_{t} \\ E_{t} \hat{y}_{t+1} \end{pmatrix} = \begin{pmatrix} -f_{y_{-}} & -f_{y_{0}} \\ 0 & I \end{pmatrix} \begin{pmatrix} \hat{y}_{t-1} \\ \hat{y}_{t} \end{pmatrix} + \begin{pmatrix} -f_{u} \\ 0 \end{pmatrix} u_{t}$

Insert the policy functions:

$$\begin{pmatrix} 0 & f_{y_{+}} \\ I & 0 \end{pmatrix} \begin{pmatrix} g_{y} \hat{y}_{t-1} + g_{u} u_{l} \\ g_{y} (g_{y} \hat{y}_{t-1} + g_{u} u_{l}) + g_{u} \underbrace{E_{t} u_{t+1}}_{=0} \end{pmatrix} = \begin{pmatrix} -f_{y_{-}} & -f_{y_{0}} \\ 0 & I \end{pmatrix} \begin{pmatrix} \hat{y}_{t-1} \\ g_{y} \hat{y}_{t-1} + g_{u} u_{l} \end{pmatrix} + \begin{pmatrix} -f_{u} u_{l} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & f_{y_{+}} \\ I & 0 \end{pmatrix} \begin{pmatrix} I \\ g_{y} \end{pmatrix} g_{y} \hat{y}_{t-1} = \begin{pmatrix} -f_{y_{-}} & -f_{y_{0}} \\ 0 & I \end{pmatrix} \begin{pmatrix} I \\ g_{y} \end{pmatrix} \hat{y}_{t-1} + \begin{pmatrix} -f_{y_{+}} g_{y} g_{u} \\ -g_{u} \end{pmatrix} u_{l} + \begin{pmatrix} -f_{y_{0}} g_{u} \\ g_{u} \end{pmatrix} u_{l} + \begin{pmatrix} -f_{u} \\ 0 \end{pmatrix} u_{l}$$

$$\begin{pmatrix} 0 & f_{y_{+}} \\ I & 0 \end{pmatrix} \begin{pmatrix} I \\ g_{y} \end{pmatrix} g_{y} \hat{y}_{t-1} = \begin{pmatrix} -f_{y_{-}} & -f_{y_{0}} \\ 0 & I \end{pmatrix} \begin{pmatrix} I \\ g_{y} \end{pmatrix} \hat{y}_{t-1} + \begin{pmatrix} -F_{u} \\ -g_{u} + g_{u} \end{pmatrix} u_{l}$$

$$\begin{pmatrix} 0 & f_{y_{+}} \\ I & 0 \end{pmatrix} \begin{pmatrix} I \\ g_{y} \end{pmatrix} g_{y} \hat{y}_{t-1} = \begin{pmatrix} -f_{y_{-}} & -f_{y_{0}} \\ 0 & I \end{pmatrix} \begin{pmatrix} I \\ g_{y} \end{pmatrix} \hat{y}_{t-1} + \begin{pmatrix} -F_{u} \\ -g_{u} + g_{u} \end{pmatrix} u_{l}$$

Schur Decomposition on Structural State-Space System

 $\begin{pmatrix} 0 & f_{y_{+}} \\ I & 0 \end{pmatrix} \begin{pmatrix} I \\ g_{y} \end{pmatrix} g_{y} \hat{y}_{t-1} = \begin{pmatrix} -f_{y_{-}} & -f_{y_{0}} \\ 0 & I \end{pmatrix} \begin{pmatrix} I \\ g_{y} \end{pmatrix} \hat{y}_{t-1}$ D $Q'TZ'\begin{pmatrix}I\\g_y\end{pmatrix}g_y\hat{y}_{t-1} = Q'SZ'\begin{pmatrix}I\\g_y\end{pmatrix}\hat{y}_{t-1}$

Multiply by Q:

 $TZ'\begin{pmatrix}I\\g_y\end{pmatrix}g_y\hat{y}_{t-1} = SZ'\begin{pmatrix}I\\g_y\end{pmatrix}\hat{y}_{t-1}$

Re-ordering of Schur decomposition

 $TZ' \begin{pmatrix} I \\ g_y \end{pmatrix} g_y \hat{y}_t$

 T_{11} and S_{11} are square matrices and contain stable Generalized Eigenvalues

 T_{22} and S_{22} are square matrices and contain unstable Generalized Eigenvalues

$$t_{t-1} = SZ' \begin{pmatrix} I \\ g_y \end{pmatrix} \hat{y}_{t-1}$$

Order stable Generalized Eigenvalues $|\lambda_i| < 1$ in the upper left corner of T and S: $\begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix} \begin{pmatrix} Z'_{11} & Z'_{21} \\ Z'_{12} & Z'_{22} \end{pmatrix} \begin{pmatrix} I \\ g_y \end{pmatrix} g_y \hat{y}_{t-1} = \begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix} \begin{pmatrix} Z'_{11} & Z'_{21} \\ Z'_{12} & Z'_{22} \end{pmatrix} \begin{pmatrix} I \\ g_y \end{pmatrix} \hat{y}_{t-1}$

Impose Stability

such that the lower (explosive) rows are always zero:

 $\begin{pmatrix} T_{11} & T_{12} \\ \mathbf{0} & T_{22} \end{pmatrix} \begin{pmatrix} Z_{11} & Z_{21} \\ Z_{12} & Z_{22} \end{pmatrix} \begin{pmatrix} I \\ g_y \end{pmatrix} g_y \hat{y}_{t-1} = \begin{pmatrix} S_{11} & S_{12} \\ \mathbf{0} & S_{22} \end{pmatrix} \begin{pmatrix} Z_{11}' & Z_{21}' \\ Z_{12}' & Z_{22}' \end{pmatrix} \begin{pmatrix} I \\ g_y \end{pmatrix} \hat{y}_{t-1}$

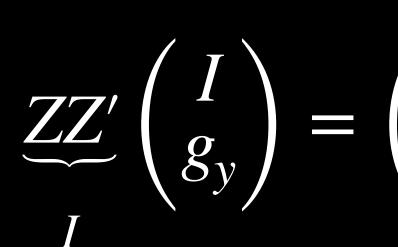
We DON'T WANT an explosive solution, so we rule this out by imposing:

 $\begin{pmatrix} Z'_{11} & Z'_{21} \\ Z'_{12} & Z'_{22} \end{pmatrix} \begin{pmatrix} I \\ g_y \end{pmatrix} = \begin{pmatrix} XXX \\ 0 \end{pmatrix}$

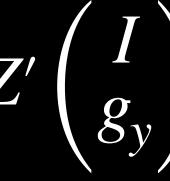
 $0 \cdot XXX + T_{22} \cdot 0 = 0 \cdot XXX + S_{22} \cdot 0 = 0$

$Z_{11} \cdot XXX + Z_{12} \cdot 0 = I \Leftrightarrow XXX = (Z_{11})^{-1}$

Focusing on the upper rows we get:



Pre-multiply by Z:



Impose Stability

$Z'\begin{pmatrix}I\\g_y\end{pmatrix} = \begin{pmatrix}XXX\\0\end{pmatrix}$

$\underbrace{ZZ'}_{g_y} \begin{pmatrix} I\\g_y \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12}\\Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} XXX\\0 \end{pmatrix}$

From the lower rows we can recover g_v :



 $\begin{pmatrix} Z'_{11} & Z'_{21} \\ Z'_{12} & Z'_{22} \end{pmatrix} \begin{pmatrix} I \\ g_y \end{pmatrix} = \begin{pmatrix} (Z_{11})^{-1} \\ 0 \end{pmatrix}$

 $Z_{12}' \cdot I + Z_{22}' \cdot g_y = 0$

 $g_y = -(Z'_{22})^{-1}Z'_{12}$

Blanchard & Khan (1980) conditions

- 1. Order condition: Squareness of Z_{22}
- 2. Rank condition: Invertibility of Z_{22} , i.e. full rank of Z_{22}



Summary

Policy function / decision rule:

Algorithm:

- 1. create *D* and *E* matrices
- 2. do a QZ/Schur decomposition with re-ordering

3.
$$g_y = -(Z'_{22})^{-1}Z'_{12}$$

4. $g_u = -(f_{y_0} + f_{y_+}g_y)^{-1}f_u$

$y_{t} = \bar{y} + g_{y}(y_{t-1} - \bar{y}) + g_{u}u_{t}$

Summary

g_v is a $n \times n$ matrix

Dynare's *oo*_. *dr*. *ghx* focuses only on states

g_{μ} is a $n \times n_{\mu}$ matrix

• only columns wrt state (predetermined and mixed) variables are nonzero;

• rows are in declaration order; rows in Dynare's *oo_.dr.ghx* are in DR order

• rows are in declaration order; rows in Dynare's *oo_.dr.ghu* are in DR order

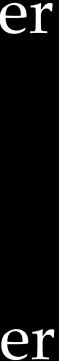


Illustration: perturbation_solver_LRE.m